Table 1 Summary of results

Signal parameter	Value at given beam current, A		
	1.0	1.5	1.7
Attenuation, dB			
Calculated	−0.85	−1.07	−1.20
Measured[a]	−0.38 to −0.84	...	...
Measured[b]	−0.30 to −0.50	−0.73 to −1.09	−1.28
Phase shift, deg			
Calculated	29.4	43.4	49.7
Measured[b]	15.5 to 21.9	28.9 to 39.1	43.7

[a] Frequency-averaged. [b] Time-averaged.

Fig. 2, there is a slight attenuation and phase shift at 0 A beam current, apparently because a low-velocity plasma diffuses out of the thruster from the discharge feed system. The attenuation and phase shift values of Fig. 2 are included in the "time-averaged" values shown in Table 1.

The standard deviation shown in Fig. 2 provides a measure of signal amplitude and phase fluctuations. In almost all cases, the standard deviation was the largest when the beam current was 1.0 A. For the case shown in Fig. 2a, the amplitude standard deviations are 1.3% and 0.9% for the 1.0- and 1.5-A cases, respectively, or about 0.1 dB in each case. The average amplitude fluctuations would have negligible effect on the performance of either the phase tracking loop or the data detection of typical spacecraft receivers.[6] The phase standard deviation of the 1.0-A case in Fig. 2b is about 1.5°, which would produce less than 0.1 dB degradation under the worst-case ratio of phase bandwidth to loop bandwidth.

Within the limited set of measured data, the attenuations and phase shifts apparently were insensitive to the rf frequency, the rf power levels, and the polarizations used in this experiment. However, more data must be taken before these effects can be evaluated accurately.

Conclusions

The results show that an S-band signal passing through an ion-thruster plume is reduced in amplitude and advanced in phase, in agreement with the simplified mathematical models. The steady-state signal attenuation levels measured in this experiment have a significant impact on the communication link performance, e.g., data rate or bit-error rate capability. In addition, multiple-thrust configurations could increase the plume plasma density, which could produce a larger signal attenuation than is experienced in the single-thruster case. The steady-state signal phase shift has no direct effect on link performance. The steady-state fluctuations in signal amplitude and phase have little effect on communication link performance at this frequency. Jumps in phase resulting from rapid changes in beam current could cause the receiver loop to lose lock, and so changes in beam current should be made gradually to allow the changing phase to be tracked by the receiver. This study confirms that the thruster plume can have a significant effect on S-band communication link performance; hence the plume effects must be considered in S-band link calculations when electric thrusters are used for spacecraft propulsion.

Acknowledgment

This paper presents one phase of research performed at the Jet Propulsion Laboratory under Contract NAS7-100, sponsored by NASA.

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Improving the Performance of Missiles by Balance Loads

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Introduction

WHEN a missile changes its direction, inertial forces act on its body. As the strength of the missile body is limited, the missile is limited in its performance, too.[1] In order to improve its performance, it is designed so that balance forces produced by a device located in the missile will operate on the missile body when it is affected by the inertial forces. These balance forces are designed to lower the effect of the inertial forces to the maximum extent.

A missile possessing the proposed system will operate better in comparison with one without it. The design of such a system for a given missile is very complicated, and serious problems have to be overcome. A somewhat unrealistic example of design of such a system in a schematic missile will clarify the basic principles of design.

Forces Acting on the Missile

It is assumed that the discussed missile changes its direction in a cycle movement in a constant radius (Fig. 1). Such a cycle movement of the missile is composed of two different movements: the overall cycle movement around the center of movement, and the self-cycle movement around the center of the mass of the missile. It is assumed that two jets put the missile into the cycle movement; the first is located in the mass center, and the second one is off center at a distance e. The jets are directed perpendicularly to the length of the missile (Fig. 1). In order to change the missile direction, the two jets have to work for a very short time Δt at a strength given by

$$F_1 = MV^2/R - JV/e\Delta tR \tag{1}$$

$$F_2 = JV/e\Delta tR$$

Received July 30, 1976; revision received Jan. 25, 1977.
Index categories: Spacecraft Configurational and Structural Design (including Loads); Structural Dynamics.

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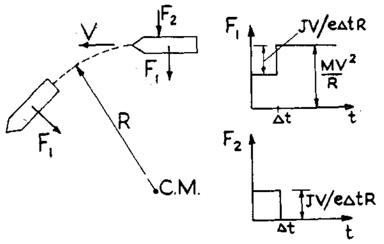

Fig. 1 Trajectory of missile and the acting forces.

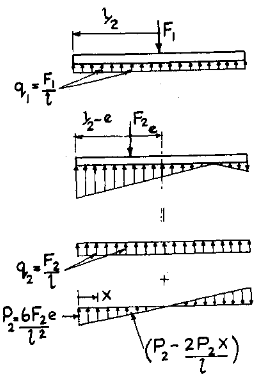

Fig. 2 The inner forces in the missile produced by the jets.

where M is the missile mass, V the missile speed, and J the moment of inertia of the missile mass. Afterwards, the first jet has to continue at a force given by

$$F_l = MV^2/R \tag{2}$$

and the second one has to stop.

The inertial forces produced in a homogeneous missile by the jets are given in Fig. 2. The inner forces in the missile body are a function of its deflection. The deflection of the missile can be calculated in coordinates located in the missile, taking into account the jet and inertial forces. It is assumed that the deflection is negligible in comparison with R, so that it has no effect on the inertial forces. The deflection y is calculated according to

$$EI\frac{\partial^4 y}{\partial^4 y} + m\frac{\partial^2 y}{\partial t^2} = F(x,t) \tag{3}$$

where E is the elastic modulus, I the moment of inertia of a cross section of the missile, m the distributed mass, and $F(x,t)$ the forces acting on the missile. It is assumed that $F(x,t)$ can be written as

$$F(x,t) = F_x(x)\ F_t(t) \tag{4}$$

The solution of Eq. (3) is given by

$$y = \Sigma Y_i(x)\ \frac{\Gamma i}{M_i w_i^2}\int_0^t w_i\ \sin w_i(t-\tau)F_t(\tau)\mathrm{d}\tau \tag{5}$$

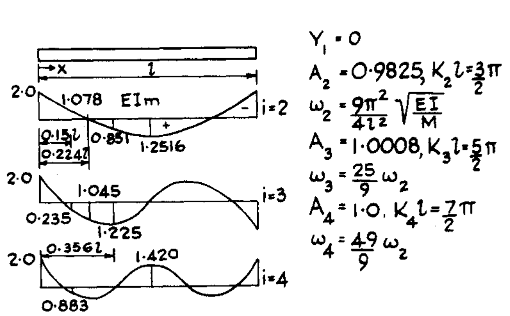

Fig. 3 Principal modes of the missile.

where

$$\Gamma i = \int_0^l Y_i(x)F_x\mathrm{d}x \text{ (generalized force for mode } i)$$

$$M_i = \int_0^l Y_i^2(x)m\mathrm{d}x \quad \text{(generalized mass for mode } i)$$

$Y_i(x)$ and w_i are the normal modes and frequencies of the homogeneous solution of Eq. (3). The normal modes and frequencies of a free beam are given in[2]

$$Y_i(x) = A_i(\sin K_iX + \sinh K_iX) - (\cos K_iX + \cosh K_iX) \tag{6a}$$

$$K_i \simeq \frac{2i-1}{2l}\pi \qquad w_i = K_i^2\frac{\sqrt{EI}}{m} \qquad M_i = ml \tag{6b}$$

The first four normal modes are given in Fig. 3. The deflection due to $F_l = MV^2/R$, which is the prominent force, is dominated by the second mode given in

$$y_2 = Y_2(x)\cdot(1.2516F_l/mlw_2^2)\ (\cos w_2t - 1) \tag{7}$$

The maximum curvature at midpoint is given in

$$\left(\frac{\partial^2 y_2}{\partial x^2}\right)x = {}^l/2 = 1.588K_2^2\frac{1.2516F_l}{mlw_2^2}(\cos w_2t - 1) \tag{8}$$

Balance Forces

The balance forces are applied on the missile by prestressed cables and gas pressure (Fig. 4). The cables are anchored at their ends to the missile structure at its axis of symmetry and attached to the side of the missile by frictionless pulleys. The cables are prestressed by jacks. This arrangement of the cables was used for the principle example only. Other alternatives in arranging the cables are possible. In practical design, there is a place for optimal analysis of the most effective arrangement, which opposes the inertial forces to maximum extent. When the cables are prestressed, the forces given in Fig. 4 act on the missile. The balance forces are in equilibrium and thus have no effect on the movement of the missile. The distribution of the balance forces B_{al} along the missile is given in

$$\begin{aligned} B_{al} = [\,&(0.3\bar{B}_l + 0.7\bar{B}_2)\cdot\delta(x=0.15l) \\ &- \bar{B}_2\cdot\delta(x=0.356l) - \bar{B}_l\cdot\delta(x=0.644l) \\ &- (0.7\bar{B}_l + 0.3\bar{B}_2)\cdot\delta(x=0.85l)\,]\cdot B_t \end{aligned} \tag{9}$$

$\delta(x=a)$ is the Dirac delta function; it is equal to 1 at $x=a$ and zero elsewhere; B_t is a function of time only. It is assumed

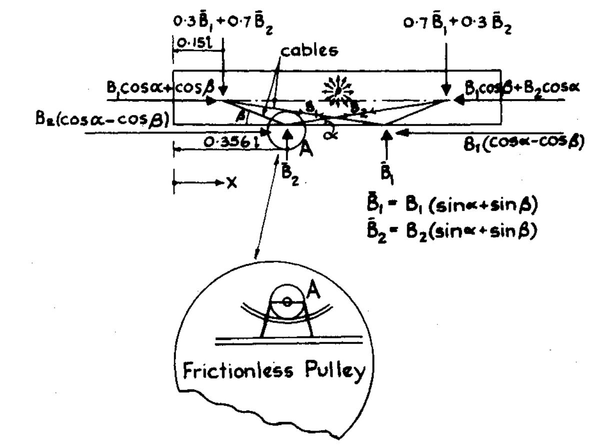

Fig. 4 Scheme of the control forces.

that the diameter of the missile is small compared with its length, and thus only the perpendicular components of the balance forces are considered. The deflection of the missile due to the jet (ignoring the axial loads), i.e., the inertial loads and balance forces, are given in

$$y=\Sigma Y_i(x)\,\frac{\Gamma_i}{mlw_i}\int_0^t \sin w_i(t-\tau)F_t(\tau)\,\mathrm{d}\tau$$

$$+\frac{\hat{\Gamma}_i}{mlw_i}\int_0^t \sin w_i(t-\tau)B_t(\tau)\,\mathrm{d}\tau \tag{10}$$

where

$\hat{\Gamma}_i=\int_0^l Y_i(x)B_x(x)\,\mathrm{d}x$ (generalized balance forces in mode i)

From Eq. (10), it is clear that, in order to cancel the effect of the first three modes, the following have to be fulfilled

$$B_t=F_t \qquad \Gamma_2=-\hat{\Gamma}_2 \qquad \Gamma_3=-\hat{\Gamma}_3 \tag{11}$$

The first equation points out that change in time of the balance forces has to be similar to the change in time of the forces acting on the missile. A special jack located in the missile is programmed to change the balance forces in time according to the change in the direction of the missile. The magnitude of the balance forces can be calculated from the last two equations of Eqs. (11), which give when $e=0.224l$:

$$F_1\cdot 1.2516-\frac{F_1}{l}\cdot\int_0^l Y_2(x)\,\mathrm{d}x$$

$$+F_2\cdot 0-\frac{F_2}{l}\int_0^l Y_2(x)\,\mathrm{d}x+\int_0^l\left(\frac{2p_2}{l}x-p_2\right)Y_2(x)\,\mathrm{d}x$$

$$=(0.3\tilde{B}_2+0.7\tilde{B}_2)1.078+\tilde{B}_2 0.851$$

$$+\tilde{B}_1 0.851+(0.3\tilde{B}_2+0.7\tilde{B}_1)1.078 \tag{12a}$$

$$F_1\cdot 0-\frac{F_1}{l}\int_0^l Y_3(x)\,\mathrm{d}x+F_2\cdot 1.045$$

$$-\frac{F_2}{l}\int_0^l Y_3(x)\,\mathrm{d}x+\int_0^l\left(\frac{2p_2}{l}x-p_2\right)Y_3\,\mathrm{d}x$$

$$=(0.3\tilde{B}_2+0.7\tilde{B}_2)(-0.235)+\tilde{B}_2 1.225$$

$$+\tilde{B}_1(-1.225)+(0.3\tilde{B}_2+0.7\tilde{B}_1)0.235 \tag{12b}$$

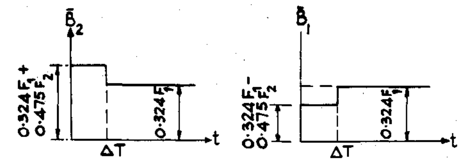

Fig. 5 Values of the control forces.

The solution is

$$\tilde{B}_1=0.324F_1-0.475F_2 \qquad \tilde{B}_2=0.324F_1+0.475F_2 \tag{13}$$

The prestressing forces are given in

$$\tilde{B}_1=\tilde{B}_1/\sin\alpha+\sin\beta \qquad B_2=\tilde{B}_2/\sin\alpha+\sin\beta) \tag{14}$$

Their change in time is given in Fig. 5. The gas pressure has no effect on the deflection, and it balances the compression of the prestressing on the missile body and has to be equal to

$$p=(B_2\cos\alpha+B_1\cos\alpha)/A \tag{15}$$

where A is the cross-sectional area, and p is constant during the whole cyclic movement.

The deflection due to the fourth mode is the dominating one when the balance loads are active. This deflection due to $F_J=MV^2/R$, which is the prominent force, when $\tilde{B}_1\times\tilde{B}_2=0.324F$, is given in

$$y_4=(\Sigma\Gamma_4/mlw_4^2)\,(\cos w_4t-1) \tag{16}$$

where

$$\Sigma\Gamma_4=(-1.42)\cdot F_1+2.0.324(0.0+0.883)F_1=0.848F_1$$

The curvature at midpoint is given in

$$\max\left(\frac{\partial^2 y_4}{\partial x^2}\right)=1.512K_4^2\frac{0.848F_1}{mlw_4^2}(\cos w_4t-1) \tag{17}$$

From Eqs. (8) and (17), it can be seen that the maximum curvature of a missile with balance forces is approximately 1/8.44 of the maximum curvature of a missile without balance forces. It is clear that a more sophisticated use of the loads can add to the resistance of the missile in an order of magnitude.

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